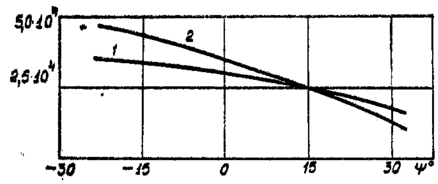

Fig. 1

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STRUCTURES AND THEIR EVOLUTION IN A TURBULENT SHEAR LAYER

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1. INTRODUCTION

From the mathematical viewpoint, turbulent fluid motion is represented by the result of exciting many strongly interacting degrees of freedom. In the motion of these degrees of freedom there is hence neither total chaos (which would permit utilization of simple statistical models), nor total coherence. Recent investigations (see e.g., [1-3]) make the idea that many turbulent flows are a system of interacting and quite stable wave packets, vortex structures, all the more likely. The spatial separateness often observed for the structures indicates that their interaction does not annihilate the possibility of considering a structure as a certain "unit" of turbulence.

There is apparently no single mechanism for the formation of structures in different turbulent flows. The widely known dissipative structures are represented by the combined product of nonlinearity and dissipation. For instance, Benard cells in convective flows and Taylor vortices in circular Couette flows originate and exist in a limited range of nonlinearity-to-dissipation ratios. In free turbulent flows, jets, wakes, and in mixing layers the dissipation plays no visible part in structure formation. It can be assumed that certain local integrals of motion are responsible for the existence of structures in these effectively nonviscous flows. The prolonged existence of structures naturally results in the idea of building up an internal statistical equilibrium therein [4-6]. As has been shown in [7, 8], isolated statistically equilibrium structures from two-dimensional point vortices

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can be formed and should exist unlimitedly in an ideal fluid. The assumption of the closeness of the structures forming turbulence to the statistically equilibrium noticeably simplifies the description and is used below in analyzing the flow in a shear layer. We hence preface this analysis by a brief exposition of the main ideas of the statistical mechanics of a system of two-dimensional point vortices [7, 8].

Motion of a two-dimensional system of n identical point vortices of intensity Γ in an unbounded space is described by a system of Hamilton equations [9]. The internal system energy, the fraction of the total kinetic energy dependent on the location of the vortices [9]

$$E = -\frac{\Gamma^2}{2\pi}\sum_{i<j}\ln|\mathbf{r}_i - \mathbf{r}_j|, \tag{1.1}$$

plays the part of the Hamiltonian, where r_i is the coordinate (two-dimensional) of the i-th vortex. It is assumed (and this assumption is confirmed by numerical computations) that the Hamiltonian vortex system relaxes to a statistical equilibrium state with the course of time, which is described by a microcanonical Gibbs distribution. It is considered that this distribution is determined completely by the integrals of motion, the coordinates of the center of vorticity $\mathbf{R} = \sum_i \mathbf{r}_i/n$, the moment of inertia, the proportional quantity [9]

$$L^2 = \frac{1}{n}\sum_i |\mathbf{r}_i - \mathbf{R}|^2, \tag{1.2}$$

and the internal energy (1.1). The quantity L governs the scale of the vortex structure under consideration. The shape of the structure is determined by a dimensionless one-particle distribution function $\tilde{P}$ relative to which an ordinary differential equation

$$\frac{1}{\eta}\frac{d}{d\eta}\left(\eta\frac{d\ln\tilde{P}}{d\eta}\right) = -4(1+\lambda) + 8\pi\lambda\tilde{P} \tag{1.3}$$

has been derived in [8], where η = r/L. Solutions of this equation are governed by a single parameter, the dimensionless temperature of the microcanonical ensemble (1/λ), or its uniquely related dimensionless internal energy E

$$E = n^2\Gamma^2\left(\tilde{E} - \frac{1}{4\pi}\ln L\right). \tag{1.4}$$

It is natural to apply this theory to describe structures in turbulent mixing layers, where they are almost two-dimensional vortices [10-12]. The pattern of shear layer evolution is represented especially graphically by the results of numerical modeling in [6], where the initial vorticity distribution in the shear layer was modeled by a system of point vortices. At each time stage the shear layer is a chain of structures within which the point vortices are organized into substructures. Cascade merging of the structures is observed, which as in the experiments in [10-12] occurs mainly in pairs. The simplest model of shear layer evolution, a linear chain of pairwise merging equilibrium structures, is examined in [6]. Neglecting completely the influence of the remianing system on the pair of merging vortices and relying on conservation of energy and the moment of inertia, the authors obtain the main evolution characteristics.

The total pattern of shear layer evolution observable in experiments and in numerical modeling is examined in this paper. Two models are studied, the simplest of which is analogous to the model in [6]. A more likely model of the merging is considered, and in particular, the influence of adjacent structures on that being formed as well as the nonconservation of the moment of momentum are taken into account. The purpose of this paper is to obtain a relationship between the fundamental shear layer parameters.

2. FORMULATION OF THE PROBLEM AND A SIMPLE MODEL

Coherent structures are ordinarily understood to be vorticity organized into coherent bunches. The velocity field is found by the known vorticity to the accuracy of a potential component, which is determined by boundary conditions. Hence, it is convenient to study the evolution of turbulent flows in terms of the vorticity distribution. In the two-dimensional problem the vorticity is similar to the charge density, and the point vortices move similarly to the two-dimensional charged particles in a strong field according to the equations of drift theory.

At the initial instant let the vorticity in an unlimited inviscid fluid be distributed uniformly in a band of width δ near the abscissa axis. It is well known that such a vorticity distribution is unstable relative to perturbations with wavelength $D \gtrsim \delta$. The time of instability development is proportional to D and the perturbation development is hierarchical in nature. Initially, a linear chain of vortex structures of the "cat's eye" type is rapidly formed because of the development of the most dangerous instability. Further evolution of the shear layer occurs by merger of these structures.

To analyze the process we use a finite-dimensional vorticity approximation. We isolate a segment of length 2A in the infinite shear layer, and replace the rest by a fixed vortex sheet of the same intensity. This vortex sheet produces an external field for the isolated layer segment and hinders its twisting into a spiral. We replace the vorticity in the isolated layer segment by a large, but finite, number M of point vortices of intensity Γ. The question of the possibility of modeling a continuous system by a system of discrete point vortices is examined in [13], where it is shown that such modeling is possible for the problem being studied. The passage to the limit $M \to \infty$, $A \to \infty$, $M\Gamma/A = \text{const}$ is performed in the final formulas. The motion of a system of M point vortices in a given external field is described by a system of Hamilton equations with the Hamiltonian [9]

$$H = -\frac{\Gamma^2}{2\pi}\sum_{i<j}\ln|\mathbf{r}_i - \mathbf{r}_j| + \Gamma\sum_i \psi(\mathbf{r}_i), \tag{2.1}$$

where ψ is the stream function of the external field. In our case

$$\psi(\mathbf{r}) = \frac{\Delta u}{4\pi}\left\{(A+x)\ln\left[\left(1+\frac{x}{A}\right)^2 + \frac{y^2}{A^2}\right] + (A-x)\ln\left[\left(1-\frac{x}{A}\right)^2 + \frac{y^2}{A^2}\right]\right\},$$

Δu is the velocity jump on the vector sheet. Because of the equation of motion the quantity H does not change in time and is later called the energy.

The interaction of the remote vortices induces the largest contribution to the magnitude of the total energy H in (2.1). It is easy to understand, however, that the effective radius of interaction is finite. The fundamental mechanism of shear layer evolution is Helmholtz instability. The scale D(t) in which instability succeeds in being developed in the time t is $D(t) \sim t$. The coherent vortex displacements $\Delta r(t) \sim t$ are also of the same order. For large distances, the logarithm in (2.1) can be represented as a series in displacements. The time-independent $\ln|\mathbf{r}_{ij}(0)|$ results in a constant in H which is not additive along the length of the layer. The field parameters of the vortex sheet are determined by the condition of conservation of the layer shape in the form of a line. Consequently, the principal part of the linear term in the displacement is mutually cancelled with the contribution of the vortex sheet. The remaining part of H is proportional to the length of the shear layer. The radius of interaction given by this part is on the order of D(t) and plays the part of a time-dependent characteristic scale of the evolutionary shear layer.

The scale D can be considered equal to the characteristic distance between the structures. The shear layer can have other characteristic scales, the radius of the structure ρ, say. However, if it is assumed that the dynamics of the layer is controlled completely by the Helmholtz instability, then all these scales should be proportional to D in the limit $t \to \infty$. This means that the shear layer is asymptotically self-similar. The occurrence of the self-similarity properties is demonstrated graphically in the models under consideration below.

The magnitude of the dimensionless miscibility parameter $\Delta = D/(2\rho)$ plays an important role in the determination of the total structure evolution pattern. If the parameter Δ is large, then as was noted in [14, 6, 15] etc., the interaction between structures is weak in strongly nonlinear systems. The structure shape is determined by its nonlinearity and depends weakly on interaction with the environment, As experiments [11] and computations [6] show, the structure diameter in the shear layer is several times less than their average separation. This permits the hope that the fundamental layer characteristics can be determined by using an expansion in the parameter Δ^{-1}. In the zeroth approximation in Δ^{-1} the

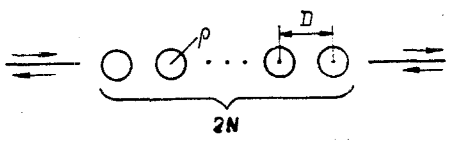

Fig. 1

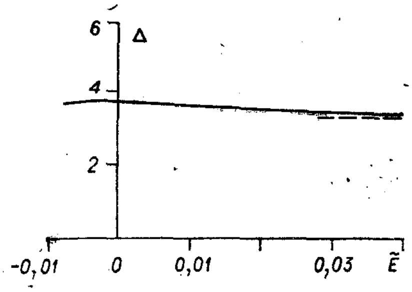

Fig. 2

interaction is considered essential only for the closures of the structures and is expressed in their merger. In the gaps between the mergers the structure interaction reduces to the mutual transfer of the centers of gravity. It is considered that the Hamilton subsystem, the vortex structure, in center of gravity coordinates is closed, and its state evolves rapidly into statistical equilibrium. Statistical equilibrium structures are determined by two parameters, the internal energy and the moment of inertia. Consequently, substantial information about the shear layer dynamics can be obtained from the conservation laws.

Let us study the result of the conservation laws in a simple shear layer model. It is assumed that after the development of the most dangerous Helmholtz instability a linear chain of structures is formed in a homogeneous shear layer. A model is considered in which the structures merge in pairs to form a linear chain of $2N_m$ structures after the m-th merger cascade (Fig. 1). The structure radius ρ_m is assumed small compared to the separation $D_m = 2^m D_0$. In this case the energy (2.1) can be written in the form

$$H = 2N_m E_m - \frac{n_m^2 \Gamma^2}{4\pi} \sum_{i \neq j} \ln D_{ij} + n_m \Gamma \sum_i \psi(R_i).$$

Here E_m is the internal energy of the structures (1.1), $R_i = D_m(i + 1/2)$, $i = -N_m, -N_m + 1, \ldots, N_m - 1$; $D_{ij} = |R_i - R_j|$; n_m is the number of point vortices in each structure. It is convenient to represent H as the sum of interaction energies of structure pairs with the numbers 2k, 2k + 1, and 2k', 2k' + 1, where k, k' run through the values $-N_m/2, -N_m/2 + 1, \ldots, N_m/2 - 1$:

$$H = \sum_k \Big\{ 2E_m - \frac{n_m^2 \Gamma^2}{2\pi} \ln D_m - \frac{n_m^2 \Gamma^2}{4\pi} \sum_{k'} \ln \{4(k' - k)^2 [4(k' - k)^2 - 1] D_m^4\} + n_m \Gamma \{\psi[(2k + 1/2) D_m] + \psi[(2k + 3/2) D_m]\} \Big\}. \tag{2.2}$$

The total energy after the pairwise mergers is written in the form

$$H = \sum_k \Big\{ 2E_{m+1} - \frac{4n_m^2 \Gamma^2}{4\pi} \sum_{k'} \ln (2|k' - k| D_m) + 2n_m \Gamma \psi[(2k+1) D_m] \Big\}. \tag{2.3}$$

The right sides of (2.2) and (2.3) contain a large non-additive contribution related to the long-range action of the Coulomb potential. As noted above, this contribution is independent of the time and is identical in (2.2) and (2.3). Hence, by equating the right sides of (2.2) and (2.3), we obtain an expression for E_{m+1} that converges in the limit as $A \to \infty$

$$E_{m+1} = 2E_m - \frac{n_m^2 \Gamma^2}{2\pi} \ln \left[D_m \prod_{p=1}^{\infty} \left(1 - \frac{1}{4p^2}\right) \right] = 2E_m - \frac{n_m^2 \Gamma^2}{2\pi} \ln \frac{2D_m}{\pi}. \tag{2.4}$$

Taking (1.4) into account, a recursion formula hence follows for the dimensionless energy

$$\frac{D_{m+1}}{2L_{m+1}} \frac{\exp(4\pi \tilde{E}_{m+1})}{\pi} = \left[\frac{D_m}{2L_m} \frac{\exp(4\pi \tilde{E}_m)}{\pi} \right]^{1/2}. \tag{2.5}$$

It is hence seen that the magnitude of the parameter D/(2L) in the limit as $m \to \infty$ is determined just by the magnitude of the dimensionless energy

$$D_\infty/(2L_\infty) = \pi \exp(-4\pi \tilde{E}_\infty). \tag{2.6}$$

The quantity L is the root-mean-square radius of the structure (see (1.2)). If the values of $\tilde{E}$ are not too large, L determines the size of the structure. For large $\tilde{E}$ the vorticity distribution becomes more and more peaked [8]. The concentration kernel of the vorticity turns out to be surrounded by an extensive rarefied "atmosphere" of point vortices. The quantity L determines the radius of this atmosphere for large $\tilde{E}$. It is hence natural to take the average radius

$$\rho = \int rP(\mathbf{r})\,d^2r$$

as the dimension of the equilibrium structure rather than the quantity L, where P(r) is the equilibrium one-particle distribution function. Correspondingly, the parameter

$$D/(2\rho) = \Delta(\tilde{E}) = \frac{\pi}{\eta}\exp(-4\pi\tilde{E}), \tag{2.7}$$

where $\eta = \rho/L$, determines the shear layer miscibility. The dependence $\Delta(\tilde{E})$ can be obtained numerically by solving (1.3). The corresponding graph is presented in Fig. 2. The dashed line denotes the asymptotic value $\Delta(\infty)$, equal to $2\sqrt{e}\approx 3.3$. The least possible $\tilde{E}$ corresponds to a tabletop vorticity distribution [8] and equals $(2\ln 2 - 1)/(16\pi) \approx -0.008$. The graph illustrates the quite weak dependence of Δ on E. The relative distinction between the maximum value of Δ and the minimum value is just 0.14. Therefore, the limit value of the miscibility parameter is determined with the relative accuracy 0.14 from just the energy conservation law.

In order to complete the calculation of the model parameters, a recursion formula must be obtained for the internal moment of inertia. The Hamiltonian (2.1) is not invariant with respect to rotation, and the total moment of inertia of the chain is not conserved. Hence, it is impossible to obtain the formula required by using reasoning similar to that utilized in deriving (2.4). We take account of nonconservation of the momentum by making the simple assumption that is based on the structure merger pattern observable in shear layers.

The structure trajectories in shear layers are qualitatively similar to point vortex trajectories in a linear chain after the loss of instability [16]. Development of the most dangerous instability results in pairwise relative rotation of the point vortices along the trajectories

$$\operatorname{ch}\frac{\pi Y}{D} - \cos\frac{\pi X}{D} = 2,$$

where X, Y are the relative coordinates of the vortex pair. The vortices come together to the minimal distance of 0.56D and then again separate. The structures merge rapidly in the shear layers, coming together to a certain minimal separation a. We assume that $a = \alpha D$, where $1 < \alpha \leqslant 0.56$, and we neglect the change in the moment of inertia of the pair during the merger itself. We equate the moment of inertia of the structure being obtained to the moment of inertia of the pair directly ahead of the merger. Using (1.2) we have

$$L_{m+1}^2 = L_m^2 + \frac{1}{4}\alpha^2 D^2; \tag{2.8}$$

$$\frac{D_m}{2L_m} = \frac{\sqrt{3}}{\alpha}\Big/\left[1 + \frac{1}{2^{2m}}\left(\frac{12L_0^2}{\alpha^2 D_0^2} - 1\right)\right]^{1/2} \xrightarrow[m\to\infty]{} \frac{\sqrt{3}}{\alpha}. \tag{2.9}$$

From (2.6) we obtain the limit value of the dimensionless energy of the structure

$$\tilde{E}_\infty = -\frac{1}{8\pi}\ln[3/(\alpha^2\pi^2)]. \tag{2.10}$$

Let us examine two cases. For $\alpha = 0.56$ $\tilde{E} = 0.0012$ and for $\alpha = 1$ $\tilde{E} = 0.047$ It is hence seen that the magnitude of a numerical parameter strongly affects the value of $\tilde{E}$ governing the shape of the equilibrium structures. However, the important parameter D/(2L) depends on α not so strongly, while the parameter Δ depends on α still more weakly (see Fig. 2).

The recursion formula (2.8) for L^2 agrees for $\alpha = 1$ with that investigated in [6]. The value of the dimensionless energy (2.10) differs several times from that obtained in [6] in this case. The difference is related to the fact that the influence of the neighbors on the merging pair was not taken into account.

3. SHEAR LAYER MODEL AS A CHAIN OF CLUSTERS

Numerical computations [6] show that vortex structures are not in complete statistical equilibrium. The representation of the vortex structure as the bound state of several equilibrium substructures is more adequate. It is easy to conceive of the possible reason for the appearance of such bound states. The instability development time in the double scale is only twice the development time of the most dangerous instabiltiy. Hence, an instability of the next order can be excited in a chain of vortex pairs before relaxation within the pair will occur, and bound clusters of three, four, and more substrates are formed. An isolated system of three vortices performs quite complicated [17, 18], and of four, stochastic motion [19]. It is natural to expect that the probability of closure and merger will increase with the increase of the number of substrates in a cluster. Hence, the number of structures in an evolutionary layer fluctuates near a certain optimal value.

Let us consider a shear layer model, a linear chain of identically oriented clusters, each of which consists of several statistically equilibrium substructures. The recursion formulas and limit relations for such a model can be derived perfectly analogously to what was done above for the simplest model. The number of substructures per cluster varies between a certain minimum and a certain maximum during shear layer evolution. The times for which the formulas are derived can always be selected in such a manner that their number would be minimal. To be specific, let us examine the case when this number is two, i.e., the next merger cycle is realized. The vortex pairs come together and clusters of four equilibrium substructures are formed. Then pairwise merger occurs within the four and pairs of the next generation form. Let the spacing between the vortices of the pair be d_m after m cascades of pairwise merging, the spacing between the clusters be D_m, and the slope of the vortex pairs to the y = 0 plane equal χ_m. The substructure radius is considered small compared with all the other dimensions. Simple but rather awkward calculations, analogous to those made above in deriving (2.5) and (2.6), yield a relationship between the structure parameters in the limit as

$$\frac{d}{2\rho}=\frac{2\Delta(\widetilde{E})}{\pi}\frac{\pi d}{D}\left[\sin^2\left(\frac{\pi d}{D}\cos\chi\right)+\mathrm{sh}^2\left(\frac{\pi d}{D}\sin\chi\right)\right]^{1/2}. \qquad (3.1)$$

After a sufficiently large quantity of mergings, the further evolution of the shear layer turns out to be self-similar for the simple shear layer models studied. The shape of the structure and the magnitude of the dimensionless parameters of the shear layer being formed after convolution of the vortex sheet can differ significantly from the self-similar. The rate at which the dimensionless parameters tend to their self-similar values is not identical for different parameters. According to (2.9), the parameter D/(2L) takes on its self-similar value of $\sqrt{3}/\alpha$ after 1-2 mergers in the simplest of the models considered, and later L increases two times for each merger. Relaxation of the parmaeter d/(2ρ) occurs somewhat more slowly. This fact should be taken into account in analyzing the data of experiments since the structures in the observed and computed mixing layers ordinarily succeed in executing just several mergings. For instance, about four merging cascades is observed in the computations [6].

Let us again stress that the interval of variation of the function Δ in (2.7) and (3.1) is narrow. This fact can also be utilized in constructing more complex shear layer and mixing layer models. The selection of a specific model should be dictated by additional information about the structure shape. It can be hoped that such information will be obtained in future experiments.

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PROPERTIES OF A MODEL FOR THE TURBULENT MIXING OF THE BOUNDARY BETWEEN ACCELERATED LIQUIDS DIFFERING IN DENSITY

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A model has been proposed [1] for the turbulent mixing of the interface between accelerated liquids differing in density, which provides solutions to various problems in analytic form. This enables one to examine the behavior of the solution in relation to the empirical constants in the model.

A more complicated model for turbulent mixing is considered here that has three parameters, and the role of the newly introduced parameter is examined. Solutions are constructed for variable acceleration given by power, step, and sinusoidal laws. It is found that the width of the mixing region can vary by up to a factor 2 in accordance with the constant in the model that characterizes the role of the inertial mechanism. A solution is obtained for the mixing of a thin layer, and the problem is referred to an integral for the case of finite thickness.

1. Model with Three Parameters. Two incompressible liquids differing in density are placed in an accelerated vessel, and the boundary between them is unstable if the acceleration is directed from the light liquid into the heavy one. This is the Rayleigh-Taylor instability. If the viscosity and surface tension are negligibly small, as occurs for high accelerations, the boundary is disrupted. One substance begins to mix with the other, and experiment shows [2] that the mixing is turbulent.

There are semiempirical models for the turbulent mixing. A very simple one with one constant was proposed in [3]. An extension of the model is given in [4, 5].

The following is a more complicated semiempirical model for turbulent mixing with three parameters for a case of two incompressible substances:

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial x} D \frac{\partial \rho}{\partial x}; \qquad (1.1)$$

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